Optimal Mesh Routing in Four-Fibre WDM Rings

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Abstract:
It is shown in a four-fibre optical WDM ring with N nodes, the number of wavelengths required to accommodate a full mesh interconnection is $(N^2-1)/8$, rounded up to the nearest integer. This result applies for all values of N, is the same with and without wavelength conversion, and represents the optimum value.

Introduction

Optical ring networks will form the next step in the rollout of WDM in the telecommunications industry. Since it is important for network planning and design to know the wavelength requirements for different traffic patterns, this Letter describes the calculation of this quantity, assuming full mesh interconnection of every node in a four-fibre ring.
There are N nodes in a ring, numbered consecutively from 0 to $N-1$. A ring has two protection fibres (which are not relevant here) and two working fibres. All paths are bi-directional and are generally carried by the two counterpropagating unidirectional working fibres [1]. Throughout, λ_N is an upper bound on the optimum number of wavelengths required to interconnect all nodes in an N-node four-fibre ring with wavelength conversion. Λ_N is similar, but it refers to a ring without wavelength conversion. Finally, μ_N is a lower bound for a ring with wavelength conversion. Clearly, $\mu_N \le \lambda_N \le \Lambda_N$.

Wavelength calculation without wavelength conversion

When performing calculations without wavelength conversion, $\lambda(i,j)$ is the number of wavelengths required to accommodate all paths of lengths i and j on the ring. Similarly, $\lambda(i)$ applies to all paths of one length.

Number of nodes a multiple of 4

Firstly, for $0<i<N/4$, $\lambda(i,N/2-i)=N/2$ for $0<i<N/4$. For some i, on a given wavelength, one would "see" the following paths on a complete circuit of the ring:

- one path of length i, followed by
- one path of length $N/2-i$, followed by
- one path of length i, followed by
- one path of length $N/2-i$, which returns to the starting point.

There are $N/2$ wavelengths in total; for each new wavelength, the "starting point" advances one node round the ring.

Since N is a multiple of four, it is also necessary to consider $\lambda(N/4)=N/4$, in which case a circuit round the ring consists of four paths of length $N/4$ laid end to end. Finally, $\lambda(N/2)=N/4$, since the "go" and "return" paths of this length between a pair of nodes may both be accommodated by a single fibre at one wavelength, where they are carried round the ring by each of the two possible routes. Each wavelength thus accommodates a pair of such paths since there are two working fibres, and since there are $N/2$ such bi-directional paths, the number of wavelengths used by them is $N/4$.

Hence the total is $\Lambda_N=\left(\frac{N}{4}-1\right)\frac{N}{2}+\frac{N}{4}+\frac{N}{4}=\frac{N^2}{8}=\left\lceil\frac{N^2-1}{8}\right\rceil$. $\lceil x\rceil$ is the smallest integer greater than or equal to x.

Number of nodes even but not a multiple of 4

As before, for $0<i<N/4$, $\lambda(i,N/2-i)=N/2$, but since N is not a multiple of 4, $1\le i\le(N-2)/4$. The layout of these paths on the ring is exactly the same as in the previous case. Since N is not a multiple of four, it is not necessary to consider $\lambda(N/4)$ as before. Finally, $\lambda(N/2)=\frac{N+2}{4}$, using a similar argument to that above.

Hence the total is $\Lambda_N=\frac{N-2}{4}\times\frac{N}{2}+\frac{N+2}{4}=\frac{N^2+4}{8}=\left\lceil\frac{N^2-1}{8}\right\rceil$ which is the same as before.

Number of nodes odd

Select two nodes x and y which are as far apart as possible i.e. the shortest path between them is of length $\frac{N-1}{2}$, and the longest path is of length $\frac{N+1}{2}$. Without loss of generality, number the nodes $0,\ldots,N-1$, numbering node x as 0 and node y as $\frac{N-1}{2}$. Using the shortest possible paths throughout, it can be shown that to interconnect x and y, and to also connect each of these nodes to all other nodes requires $\frac{N-1}{2}$ wavelengths. To demonstrate this, suppose the wavelengths are numbered $1,\ldots,\frac{N-1}{2}$. Then for $i=1,\ldots,\frac{N-1}{2}$, the following paths are set up on wavelength i:

- node 0 is connected to node i,

- node $\frac{N-1}{2}$ is connected to node $\frac{N-1}{2}+i$, and
- node $\frac{N-1}{2}+i$ is connected to node 0.

Also, for $j=1,\ldots,\frac{N-1}{2}-1$, node j is connected to node $\frac{N-1}{2}$ on wavelength j.

To complete the full interconnection of all N nodes, the remaining $N-2$ nodes must be fully interconnected, requiring Λ_{N-2} wavelengths. Hence $\Lambda_N = \frac{N-1}{2}+\Lambda_{N-2}$. Since $\Lambda_3 = 1$,

$$\Lambda_N = \frac{N-1}{2}+\frac{N-3}{2}+\frac{N-5}{2}+\cdots+2+1=\frac{\frac{N-1}{2}\times\frac{N+1}{2}}{2}=\frac{N^2-1}{8}=\left\lceil\frac{N^2-1}{8}\right\rceil$$

This is the same result as before.

Combined result

The result for all the above cases can be expressed as $\left\lceil\frac{N^2-1}{8}\right\rceil$. This is the number of wavelengths required to connect every node in a ring to every other node without wavelength conversion. Both Elrefaie [1] and Hill [2] have quoted a similar result, but only for networks with an odd number of nodes and with wavelength conversion. The novelty of this work lies in the fact that the results have been shown to apply to a much wider scenario. The optimality of this result is now demonstrated.

Optimality of the results

The lower bound μ_N on the optimum number of wavelengths required when wavelength conversion is considered can be derived by adding up the lengths of all paths and dividing by N. For N even, there are N paths each of length $1,\ldots,N/2-1$, and $N/2$ paths of length $N/2$:

$$\mu_N = \left\lceil\frac{N\sum_{i=1}^{N/2-1} i+\frac{N}{2}\times\frac{N}{2}}{N}\right\rceil=\left\lceil\frac{N}{4}\left(\frac{N}{2}-1\right)+\frac{N}{4}\right\rceil=\left\lceil\frac{N^2}{8}\right\rceil=\left\lceil\frac{N^2-1}{8}\right\rceil$$

For odd N, there are N paths each of length $1,\ldots,\frac{N-1}{2}$:

$$\mu_N = \left\lceil\frac{N\sum_{i=1}^{\frac{N-1}{2}} i}{N}\right\rceil=\left\lceil\frac{\frac{N-1}{2}\times\frac{N+1}{2}}{2}\right\rceil=\left\lceil\frac{N^2-1}{8}\right\rceil$$

So for all N, $\mu_N=\Lambda_N$, and since $\mu_N\le\lambda_N\le\Lambda_N$, then $\mu_N=\lambda_N=\Lambda_N$ i.e. the results are optimal and apply both with and without wavelength conversion.

Conclusions

In a mesh-configured four-fibre ring, an optimal wavelength path configuration has been found both with and without wavelength conversion, which requires $\left\lceil \frac{N^2-1}{8} \right\rceil$ wavelengths; wavelength conversion offers no advantage in terms of wavelength use. This number of wavelengths is roughly proportional to the square of the number of nodes; for examples, see Table 1. A simple algorithm for the wavelength allocation is implicit in the discussion: it offers an easy way to plan wavelength use in a practical ring.

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References

1. A. F. Elrefaie: "Multiwavelength Survivable Ring Network Architectures", *ICC '93*, Geneva, Switzerland, May 23-26, 1993
2. G. R. Hill: "A Wavelength Routing Approach to Optical Communications Networks", *INFOCOM '88*, Ch. 127, 1988, pp354-362

Number of nodes, N	Number of wavelengths, $\left\lceil \frac{N^2-1}{8} \right\rceil$
1	0
2	1
3	1
4	2
5	3
6	5
7	6
8	8
9	10
10	13
11	15
12	18
13	21
14	25
15	28
16	32

Table 1: Number of wavelengths required for mesh connections in a four-fibre WDM ring.